KINETICS OF PHYSICAL PROCESSES IN PINCH-DISCHARGE LASERS

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The kinetics of physical processes in pinch-discharge lasers is investigated. An analytic solution is proposed for the relaxation problem of the population of discrete levels of ionized argon (ArII) in rapid compression of a high-temperature nonequilibrium plasma. An expression for population inversion as a function of the parameters of the plasma and unit is obtained on the basis of the solution and also of a simplified model for the compression of a plasma column. Basic experimental dependences for pinch-discharge lasers are analyzed.

1. Ion generation in a pinch discharge has been previously obtained and investigated [1-7]. A detailed survey of the experimental results can be found in [4]. Pinched lasers yield significant generation power per pulse, and afford advances in the field of short wavelengths. To estimate the future state and possibilities of such lasers and also to improve and optimize them it is necessary to study the kinetic processes that result in generation. This set of problems has not been developed, nor has the mechanism by which population inversion is created been clarified.

The purpose of this study was to investigate the kinetics of physical processes in pinched lasers and to explain the basic experimental dependences.

The physical cause for the formation of population inversion is that the radiative decay probability of the lower laser level is greater than at the upper level. This leads to a repopulation of the upper laser level in ArII ionization by an electronic shock.

A complete analysis of relaxation phenomena in pinch discharges would be quite complex. However, the problem can be reduced by making a number of simplifications to the analytic method of finding the population distribution function for discrete levels of ArII, which has been previously set forth [8], and generalizing it to a case of rapid variation in plasma density. It is possible to obtain the experimental characteristics of a laser by constructing the corresponding distribution function, which determines the plasma parameters, and by knowing how these parameters vary.

We will consider the population distribution function for discrete levels of ArII under rapid plasma compression conditions. The population kinetics of the discrete levels is considered for a slowly varying (in comparison with the relaxation time) electron temperature T_e as the plasma density rapidly varies. We will assume the plasma to be optically thin and spatially homogeneous. Then the system of population balance equations has the form

$$\frac{dN_{nl}}{dt} + N_{nl}\,\mathrm{div}\,\mathbf{v} = N_e \sum_{n'l' \neq nl} [V(n'l', nl)\,N_{n'l'} - V(nl, n'l')\,N_{nl}] + \sum_{E_{n'l'} > E_{nl}} A(n'l', nl)\,N_{n'l'} - \sum_{E_{n'l'} < E_{nl}} A(nl, n'l')\,N_{nl} \quad \frac{dN}{dt} + N\,\mathrm{div}\,\mathbf{v} = 0 \tag{1.1}$$

Here, N_{nl} and E_{nl} is the population and energy of the nl-th ArII level (n is the principal, and l the orbital, quantum number), N_e is the density of electrons, N is the total density of heavy plasma particles, $V(nl, n'l')$ is the ion–electron inelastic collision probability, averaged over the Maxwell electron distribution, $A_{nl,n'l'}$ is the radiative decay probability of the levels, and v is the hydrodynamic plasma velocity. By virtue of spatial homogeneity, the total derivative $d/dt = \partial/\partial t + (v\nabla)$ is equal to the partial derivative $\partial/\partial t$ with respect to time.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 14-21, July-August, 1974. Original article submitted July 9, 1973.

To solve the system (1.1) of equations for the population N_{nl} we normalize to the total density of heavy plasma particles, assuming the electron temperature T_e to be an independent parameters determining the population N_{nl},

$$N_{nl} = N\overset{\circ}{f}_{nl}x_{nl}, \quad \overset{\circ}{f}_{nl} = g_{nl}\exp\left(-\frac{E_{nl}}{T_e}\right) \tag{1.2}$$

where g_{nl} is the statistical weight of the level nl.

Substituting Eq. (1.2) in (1.1) and using the detailed-balance principle for the direct and inverse processes, we find

$$\overset{\circ}{f}_{nl}V(nl, n'l') = \overset{\circ}{f}_{n'l'}V(n'l', nl) \tag{1.3}$$

and a free relaxation equation for the dimensionless reduced populations x_{nl},

$$\frac{dx_{nl}}{dt} = N_e\sum_{n'l'\neq nl}V(nl, n'l')(x_{n'l'} - x_{nl}) + \sum_{E_{n'l'}>E_{nl}}A(n'l', nl)\frac{\overset{\circ}{f}_{n'l'}}{\overset{\circ}{f}_{nl}}x_{n'l'} - \sum_{E_{n'l'}<E_{nl}}A(nl, n'l')x_{nl} \tag{1.4}$$

We will use a single-quantum approximation to solve these equations, assuming that the transitions satisfy the selection rule

$$n' = n,\ n\pm 1, \qquad l' = l\pm 1 \tag{1.5}$$

The radiative decay of the levels will be taken into account by the relationships

$$\begin{gathered}\sum_{E_{n'l'}<E_{nl}}A(nl, n'l') \to A(nl, n-1, l-1) + A(nl, n-1, l+1) = A(nl)\\ \sum_{E_{n'l'}>E_{nl}}A(n'l', nl)\frac{\overset{\circ}{f}_{n'l'}}{\overset{\circ}{f}_{ul}}x_{n'l'} \to A^*(n+1l)x_{n+1,l}\\ A^*(n+1l) = A(n+1l)\frac{\overset{\circ}{f}_{n+1,l}}{\overset{\circ}{f}_{nl}}\end{gathered} \tag{1.6}$$

We assume a Boltzmann distribution among each three neighboring levels ($x_{n,l-1} = x_{n,l} = x_{n,l+1}$). The system (1.4) decomposes, by taking into account Eq. (1.5) and (1.6), into a system of balance equations for the populations of discrete levels with specified values of the orbital number l = 0, 1, 2, ..., that is

$$\begin{gathered}\frac{dx_{nl}}{dt} = N_eR_{n,n+1}(l)(x_{n+1l} - x_{nl}) + N_eR_{n,n-1}(l)(x_{n-1l} - x_{nl}) - A(nl)x_{nl} + A^*(n+1l)x_{n+1l}\\ R_{n,n+1}(l) = V(nl, n+1\,l-1) + V(nl, n+1\,l+1)\\ R_{n,n-1}(l) = V(nl,\ n-1\ l-1) + V(nl,\ n-1\ l+1)\end{gathered} \tag{1.7}$$

Equations (1.7) as applied to ArII where the principal quantum number takes the values n = 3, 4, ..., can be written in a different form suitable for further calculations, namely

$$\begin{gathered}x_{nl} = x_{3l} + \sum_{m=3}^{n-1}\frac{1}{R_{m,m+1}(l)}\left(\varphi_m + \sum_{i=4}^{m}\varphi_{i-1}\prod_{k=i}^{m}\frac{R_{k,k-1}(l)}{R_{k-1,k}(l)}\right)\\ \varphi_m = \frac{1}{N_e}\frac{dx_{ml}}{dt} + \frac{A(ml)}{N_e}x_{ml} - \frac{A^*(m+1l)}{N_e}x_{m+1l}\end{gathered} \tag{1.8}$$

Equation (1.8) is a recursion equation in x_{nl} to a first-order approximation for the derivative of the population of the ground state (for details, see [8])

$$\dot{x}_{ml} = \dot{x}_{3l}$$

Expanding it, and assuming that the detailed balance principle (1.3) also holds for the probabilities $R_{k,k-1}(l)$ and $R_{k-1,k}(l)$ we find

$$\begin{gathered}x_{nl} = Q_3^n(l)x_{3l} + \sum_{m=3}^{n-1}\frac{\overline{Q_m^n(l)}}{\overset{\circ}{f}_{ml}R_{m,m+1}(l)}\sum_{k=3}^{m}\overset{\circ}{f}_{kl}x_{3l}^{(1)}\\ x_{3l}^{(1)} = \frac{1}{N_e}\frac{dx_{3l}}{dt}, \quad n = 4, 5, \ldots, \quad l = s, p, d\\ Q_m^n(l) = \prod_{k=m}^{n-1}\left(1 + \frac{A(k+1l)}{N_eR_{k+1,k}(l)}\right)^{-1}, \quad Q_3^3 = 1\end{gathered} \tag{1.9}$$

Since the 3p level in ArII is a lower level and pumping at the 4f, fp and 4d levels basically proceeds from the 3p state, the subscript $3l$ must be replaced by 3p in Eq. (1.9) and it is necessary to set $R_{3,4}(l) = R_{3,4}(p)$.

We will consider as an example the expression for population inversion between the 4p and 4s configurations

$$\frac{\Delta N}{g}=\frac{N_{4p}}{g_{4p}}-\frac{N_{4s}}{g_{4s}}=N_{3p}\left(1+\frac{\varkappa}{R_{3,4}(p)}\right)\left[\left[1+\frac{A(4p)}{N_eR_{4,3}(p)}\right]^{-1}\exp\left(-\frac{E_{4p}}{T_e}\right)-\left[1+\frac{A(4s)}{N_eR_{4,3}(s)}\right]^{-1}\exp\left(-\frac{E_{4s}}{T_e}\right)\right] \quad (1.10)$$

$$\varkappa=\frac{1}{N_e}\frac{dx_{3p}}{dt}\frac{1}{x_{3p}}=\frac{1}{N_e}\frac{d}{dt}\ln\frac{N_{3p}}{N} \quad (1.11)$$

It is evident from Eq. (1.10) that inversion between the 4p and 4s levels is not observed in the case of purely collision-like relaxation.

The final expression for the distribution function and the population inversion can be obtained if we know the actual time dependences for the population of the ground state of ArII and the electron density of the plasma. We will assume that the populations of the upper levels are smaller than at the lower levels when ArII is ionized. In this case, the parameter $\varkappa$ is determined by setting the population of the (n_0+1)-th discrete level equal to 0

$$\varkappa^{-1}=-\sum_{m=3}^{n_0}\frac{1}{\overset{\circ}{f}_{mp}R_{m,m+1}(p)\,Q_3^m(p)}\sum_{k=3}^{m}\overset{\circ}{f}_{kp} \quad (1.12)$$

As a rule, the probability $R_{3,4}(p)$ is one-twentieth to one-tenth $R_{4,5}(p)$, so that we may set $\varkappa=-R_{3,4}(p)$ with a sufficient degree of accuracy.

Suppose a plasma consists of ArII and ArIII with densities N_1 and N_2, respectively. Then using the charge conservation condition and the expression for the total density of heavy plasma particles

$$N_e=N_1+2N_2, \quad N=N_1+N_2 \quad (1.13)$$

and also assuming that $N_{3p}\approx N_1$, Eqs. (1.11) and (1.13) can be integrated together. We obtain

$$N_{3p}\approx N_1=2\eta N\left[\eta+\exp\left(2|\varkappa|\int_0^t N(t')\,dt'\right)\right]^{-1}$$
$$N_e=2N\left[1+\eta\exp\left(-2|\varkappa|\int_0^t N(t')\,dt'\right)\right]^{-1}, \quad \eta=\frac{N_{10}}{N_{20}} \quad (1.14)$$

where N_{10} and N_{20} are the initial ArII and ArIII densities.

2. Let us consider the physical processes and the generation mechanism in a pinch discharge. Population inversion appearing in a pinch discharge depends, in accordance with Eqs. (1.10) and (1.11), on the plasma parameters N_e, T_e, and N and the initial conditions. To determine these parameters and their time dependences, it is necessary to know the dynamics of a pinch discharge. The large sizes of the discharge current ($I=10^3$-10^6 A) and its high rates of growth ($\dot{I}=10^{10}$-10^{12} A/sec) are the distinctive feature of such a discharge.

The experimental data obtained allow us to divide the process by which a pinch discharge forms into three stages. At the electrical arcover stage, the electron temperature and plasma conductivity grow, and the magnetic field is driven from the control part of the discharge (a skin shell is formed). The plasma column shrinks to the discharge axis as a result of the interaction of the discharge current with its proper magnetic field at the stage of compression. Here, the electron temperature can be considered practically invariant. The last stage of a pinch discharge is characterized by thermalization of the plasma's ion component and by the fact that electron and ion temperatures are equal. At this stage the electron temperature rapidly grow.

In the general case, the compression dynamics of a plasma column is described by means of a complex system of equation from magnetohydrodynamics. We will use a previously [k, 10] set-forth model for plasma description to classify the basic laws associated with the creation of population inversion. Within the framework of this model the volume of a gas having a cylindrical form is compressed by a skin shell when an expressed skin effect is present. An expression was obtained [9] for the compression time of a plasma column, assuming that the current increases by the linear law $I=\dot{I}t$,

$$t_1\approx a_0(MN_0)^{1/4}c^{1/2}\dot{I}^{-1/2} \quad (2.1)$$

where a_0 is the initial radius of the gas-discharge chamber, N_0 is the initial gas density, M is the mass of a heavy gas particle, c is the speed of light, and $\dot{I}$ is the rate at which the discharge current increases.

The compression process of a plasma column has been previously investigated [11] in the absence of a magnetic-field skin effect, where it was shown that the compression time for a power law of growth of the current ($I = I^{(n)}t^n$) is determined by the expression

$$t_1 \approx \left(\frac{a_0^2 c\,(MN_0)^{1/2}}{I^{(n)}}\right)^{1/n+1} \tag{2.2}$$

Equation (2.2) when n = 1 coincides with Eq. (2.1).

Let us consider population inversion between the 4p and 4s levels at the stage at which plasma column compresses, namely when the electron temperature weakly depends on time and can be considered practically constant. Limiting ourselves to the first two terms in the sum (1.12) we have

$$\frac{\Delta N}{g} = N_{3p}\left\{1 - \left[N_e + \frac{A\,(4s)}{R_{4,3}\,(s)}\right]^{-1}\left[N_e + \frac{A\,(4p)}{R_{4,3}\,(p)}\right]\right\}\frac{R_{3,4}\,(p)}{R_{4,5}\,(p)}\left(1 + \exp\left(-\frac{E_{4p}}{T_e}\right)\right) \tag{2.3}$$

As can be seen from this equation, population inversion is entirely determined by the electron temperature for small values of the electron density. In the case of high electron densities, the inversion between the levels 4p and 4s can vanish since relaxation becomes collision-like. The critical values of the electron density are determined by setting Eq. (2.3) equal to zero,

$$(N_e)_{\max}\left(\exp\frac{\Delta E}{T_e} - 1\right) = \frac{A\,(4s)}{R_{4,3}\,(s)} - \frac{A\,(4p)}{R_{4,3}\,(p)}\exp\frac{\Delta E}{T_e} \tag{2.4}$$

The value of the limiting electron density was found equal to $5 \cdot 10^{16}$ cm^{-3} for T_e = 4.3 eV = $5 \cdot 10^4$°K and previously [13] given values for the radiative decay probabilities of the levels. Here and below, probabilities for inelastic ArII-electron collisions previously [12] given are used in the calculations.

For the electron densities

$$N_e \ll \frac{A\,(4s)}{R_{4,3}\,(s)} \tag{2.5}$$

population inversion is equal to the population of the upper laser level. As is evident from the foregoing, the basic pattern by which inversion appears has much in common with the case of an ordinary ArII laser [13, 14]. Excited ArII levels are populated due to electron shocks. Purification of the lower working level is caused by a radiative mechanism. The greater is the electron temperature and population of the ArII ground state, the greater is the ionization flow to the upper levels from the ground state and the greater is the population of the excited levels. States with negative temperature are possible in a plasma of singly and multiply ionized argon atoms at low electron densities when the radiation flux emptying the upper excited levels is comparable in magnitude with the ionization flow.

Results are presented below of a calculation of the population inversion as a function of time, electron temperature, and initial gas densities. To qualitatively represent the time dependence of the population inversion process, we will consider the variation in the plasma density over time as given by the law

$$N = N_0\left(\frac{N_m}{N_0}\right)^{t/t_1} \tag{2.6}$$

where N_m is the plasma density at the moment of maximum compression of the plasma column.

Figure 1 depicts the dependence of population inversion $\Delta N/g$ on time for T_e = 4.3 eV, $N_0 = 3 \cdot 10^{14}$ cm^{-3}, $N_m = 4 \cdot 10^{15}$ cm^{-3}, a_0 = 10 mm and I = 10^{10} A/sec. The initial increase in population inversion is explained by the increase in plasma density as a consequence of compression of the current shell to the center while its full is explained by further ionization of the singly ionized argon. The position of the maximum on the curve depends on the ratio between compression time and characteristic ArII ionization time. The most favorable conditions for generation are found for the case when ArII cannot successfully strongly ionize, i.e., for low compression times or low current growth rates. It can be shown by investigating the population of the ground state that population inversion is at a maximum at the moment $t = t_1$ if

$$\frac{1}{N}\frac{dN}{dt} > 2\,|\varkappa|\,N \tag{2.7}$$

In fact, when $t > t_1$ a cumulation stage begins at which transmission of thermal energy from the ions to the electrons results in the temperature of the latter sharply growing, which finally leads to acceleration

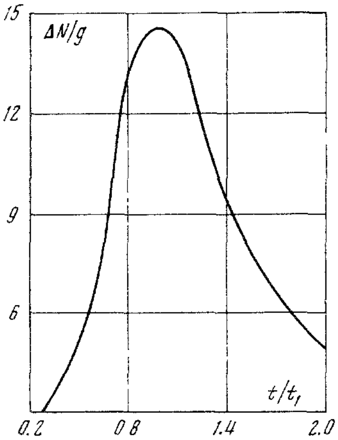

Fig. 1

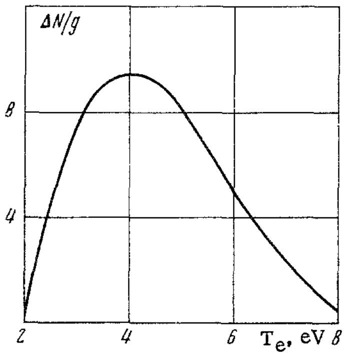

Fig. 2

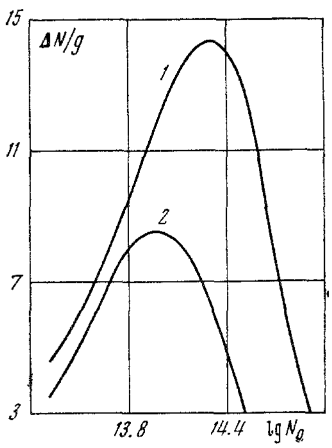

Fig. 3

of the ArII ionization process, and to an interruption in generation. Moreover, generation can be cut short if ion temperature rapidly grow, increasing Doppler broadening. Thus ArII generation at the cumulation moment is rapidly cut short, though ion generation may appears at higher ionization multiplicity.

Figure 2 depicts the maximum population inversion at the moment $t = t_1$ (it is assumed that condition (2.7) holds) as a function of the electron temperature T_e given the above parameters of the unit. At low T_e the excitation probabilities of the upper laser levels $R_{3,4}(p)$ grows faster than the probability $R_{4,5}(p)$ at which it empties, at the same time remaining small in magnitude (and scarcely vary the populations N_{3p} of the ground state). This is the reason for the growth in population inversion at low T_e. In the case of large T_e the fall $\Delta N/g$ is explained by ArII ionization and by the decrease in the population of the ground state.

The number of active generation centers increases with increasing initial gas densities, though ArII ionization proceeds more rapidly and decreases the population of the ground state, so that the dependence of $\Delta N/g$ on N_0 reveals a maximum.

If we analyze Eq. (1.4) at the extremum for $t = t_1$, assuming that plasma density at the moment N_m of maximum compression as a function of the initial gas densities has the form

$$N_m = BN_0^{\alpha}, \quad B = B(\alpha, a_0, \dot{I}, \ldots) \tag{2.8}$$

and if we also assume that

$$\int_0^{t_1} N dt \approx N_m t_1$$

we find a formula for the initial gas densities at which the population of the ground state is at a maximum, and, assuming Eq. (2.5) we have for the population inversion,

$$(N_0)_{opt}^{\alpha+1/4} = \frac{z_0 \dot{I}^{1/2}}{2|\varkappa| B a_0 M^{1/4} c^{1/2}} \tag{2.9}$$

where z_0 is the root of the equation

$$1 + \exp(-z) = \frac{\alpha + 1/4}{\alpha} z$$

We find a value for $(N_0)_{opt}$ of $2 \cdot 10^{14}$ cm^{-3} ($P_0 \approx 10^{-3}$ mm Hg) for $\alpha = 3/4$ and the given parameters [4] of the unit. The value of the constant B is found from the condition $N_0 = 3 \cdot 10^{14}$ cm^{-3} $N_m = 4 \cdot 10^{15}$ cm^{-3}.

Figure 3 gives curves for the dependence of $\Delta N/g$ on the initial gas density N_0 for two values of the gas discharge tube radius ($a_0 = 1$, 2 cm) (curves 1 and 2, respectively), while $\alpha = 3/4$ is independent of a_0 for a constant B. The optimal initial densities fall with increasing gas-discharge tube radius, and the maximum value of the population inversion decreases, which is in accord with the experimental data. These curves are true for low initial densities, satisfying Eq. (2.7). If condition (2.7) does not hold, it becomes difficult to investigate the population inversion maximum as a function of a_0 due to the need for determining the moment of time at which $\Delta N/g$ is at a maximum and also due to the absence of any dependence between B and a_0.

Population inversion increases and the generation duration decreases with increasing rate of current growth. I cannot increase without limit since the plasma density N_m would become so great with increasing current growth rate that relaxation becomes purely collision-like.

The mechanism by which generation is obtained in the case of ions of high multiplicity under pinch discharge conditions at the compression stage will be the same as in the case of ArII. Inelastic ion–

electron collision probabilities become less at optimal initial densities greater in passing to ions with high ionization multiplicity. The growth in the radiative probabilities increases the negative absorption coefficient and also increases the limiting value of the electron density at which relaxation is not longer a collision phenomenon.

To qualitatively analyze ion generation at the cumulation stage, it is necessary to take into account the term $\partial f_{nl}{}^{\circ}/f_{nl}{}^{\circ}\partial t$, caused by the rapid growth in the electron temperature. A number of qualitative features of the analysis conducted above are preserved in this case.

The chief directions for increasing the efficiency of pinched lasers include the creation of high multiplicity ion generation (preionization is necessary in this case), the increase in the repetition frequency (constraints due to a hot system), and a study of the possibilities of a collision mechanism for purifying the lower working level.

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